Tutorial Assignment 4: Superfluids and Superconductors

Write clearly. Answer to 4 significant figures. Marks given for units and explanation of calculations.

1. There is 1 mol of boson gas in a box at temperature T.

   i) State the formula for the number of particles in a state with energy ε. What is the name of this formula? [2]

   ii) The chemical potential μ is negative. Sketch a graph and explain why. [2]

   iii) Derive an integral expression for the number of bosons N. [2]

   iv) When T falls, μ changes. Explain how μ changes – using a graph. Using the same graph, explain what happens to μ when T approaches zero. [2]

   v) State the solution for the integral in (iii) when $\mu = 0$. Why is it possible for this solution to be less than N? Using (iii), suggest a physical explanation. [2]

   vi) Denote the solution in (v) by N_{ex}. When T falls, μ approaches zero. Find N_{ex} when μ first reaches zero. [2]

   vii) Suppose that the bosons are ^{4}He atoms, and the volume of 1 mol is 27.58 cm^3. When μ first arrives at zero, find the temperature T_{BE} (using (v)). What is the physical significance of T_{BE}? [2]

   viii) Derive an integral expression for the total energy above T_{BE}. Derive also an integral expression for the total energy below T_{BE}. [2]

   ix) Write down the solution for the second integral in (viii). Find the heat capacity for 1 mol of ^{4}He at T_{BE}. [2]

   x) Sketch a graph of C against T. Show how to obtain the curve below T_{BE}, the curve above T_{BE} and the peak height. How may this explain superfluid ^{4}He? [2]

2. It is possible for two electrons to attract each other with the help of positive ions in a metal.

   i) One mole of niobium is 10.84 cm^3. Find its Fermi energy. How fast is an electron at the Fermi level? [2]

   ii) Sound travels at 3480 m/s in niobium. Find the Debye frequency. Find the time that a disturbed positive ion takes to go back to its original position. [2]

   iii) The electron in (i) leaves behind a trail of displaced ions because of (ii). How long is this trail? [2]

iv) The niobium atoms are d = 3.3 Å apart. How long is the trail in terms of d? What does this tell us about the two attracting electrons? [2]

v) Suppose that an electron is at the midpoint of the line joining two ions separated by d. With what force does the electron attract each of the ions? [2]

vi) Because of screening by other electrons, this attraction is only felt within a range of about d. How long does a Fermi level electron take to travel this distance? [2]

vii) Using (v) and (vi), find the impulse on the positive ion due to the electron. At what speed does the ion move as a result of this impulse? (relative atomic mass of niobium = 93)

viii) Using (ii), find the displacement of the positive ion in (vii). Then find the resulting change in potential at the midpoint between two neighbouring ions.

ix) Using the above results, sketch the potential well of the two attracting electrons. Label the length and depth of the well. Given T_c = 9.3 K, estimate the binding energy due to this well. [2]

x) What fraction of the kinetic energy of each of the electrons is the binding energy? What should happen to two particles if kinetic energy is more than potential energy? Why does this not happen to the two electrons? [2]